
QUANTAL RISK MODEL FATQ'S

Frequently Asked Technical Questions

Who is Quantal?

Quantal was founded in the mid 1990's with the goal of providing an accurate, global equity risk model based on daily returns data. Its flagship product, Quantal PRO (Portfolio Risk and Optimization), is a front-office application used by institutional money managers, hedge funds, and pension plans worldwide for portfolio assessment and optimization. Quantal has since expanded its risk model to include fixed income and derivatives and is recognized as an industry leader in risk.

Quantal recently partnered with Thomson/Reuters to integrate its risk model with Thomson/Reuter's JRisk/JROD product.

How often does Quantal update its risk estimates?

Quantal updates its risk estimates every night to provide the most responsive and timely model possible. Even if, say, the US markets are closed for a trading holiday, US security risk estimates are updated to reflect their correlations with foreign securities.

Which asset classes are covered by Quantal?

The Quantal risk model covers global equities, fixed income, currencies, and precious metals. Equity ETF's are also covered. Derivatives on equities, bonds, and interest rates are replicated using the underlying instruments.

What does Quantal use for its estimation universe?

Quantal uses an estimation universe consisting of nearly 6500 global securities intended to represent the entirety of its coverage. The model first

computes risk estimates on the estimation universe and then applies linear regression to compute risk estimates for the remainder of the coverage universe.

What is Quantal's global security coverage?

Quantal covers more than 23,000 global securities in all countries with an established market. Any equity which is not already in Quantal's coverage universe can be added provided there is sufficient liquidity to provide an accurate time series of daily closing prices.

How does Quantal handle risk estimates in different base currencies?

Currently, Quantal calculates risk estimates for five different base currencies: Australian dollar, Canadian dollar, British pound, Japanese yen, and US dollar. The risk estimates for every security are generated separately in each of the five base currencies so that they accurately reflect the local currency risks.

Can Quantal run country-specific models?

Yes, Quantal has the data and technology to run country-specific models should a client require this.

How does Quantal handle corporate actions?

Mergers: An historical return series is prepared for the merged company, based on a market cap weighted average of the returns of the two individual companies that merged.

Acquisitions: The acquired company ceases to exist, so after the deal closes risk estimates are no longer provided for that stock. The returns data used to

prepare the risk estimates of the acquiring company are based on the historical returns for the acquiring company.

IPOs: For IPOs there is no historical return data available. Quantal has developed a technique to fill in the missing returns with appropriate data based on the characteristics of the company that completed an IPO (for example, country of origin, industry, etc). Initially these returns are used to prepare the risk estimates, but as actual returns become available for the IPO, the actual returns will be used.

Spinoffs: Quantal considers the nature of the company being spun off. If it is similar to the parent company, we use the parent company's historical returns. If we determine the historical returns would not be representative, then the spinoff is handled in the same way as IPOs.

What is Quantal's implicit factor model?

The Quantal implicit multiple factor model is specified as:

$$R_{it} = \alpha_i + \beta_{i1}F_{1t} + \dots + \beta_{iK}F_{Kt} + \epsilon_{it}$$

where R_{it} is the return on security i , $i = 1, \dots, N$, in period t , and F_{kt} , $k = 1, \dots, K$, are pervasive factors affecting the security returns. Quantal's model is a conditional model; the factor loadings β_{iK} and return volatilities are computed daily, so in principle they bear a t -subscript.

Portfolio risk can be interpreted and controlled in terms of conditional measures of risk like conditional volatility and tracking error, or it can also be interpreted and controlled by attributing (ex ante) the risk to cross-sectional characteristics of stocks, such as industry, country, style, size, etc. In Quantal's approach, a portfolio's exposure to these latter attributes is assessed from the conditional risk structure, not from estimating regressions over long periods of history.

Standard techniques for computing the factors are well known in multivariate statistics. Broadly, they include factor analysis, principal components, and cluster analysis. One way or another, all the techniques try to identify "clusters" of assets whose returns tend to move together. It is a strength rather than weakness of the method that these clusters often don't line up with, say, industry classifications

because these classifications are generally quite poor; indeed stock price behavior figures large in investors' everyday identification of groups of stocks such as "Internet stocks."

What is PCA (principal components analysis)?

Quantal's implicit factor model uses principal components analysis (PCA), a standard technique in linear algebra, statistics, and regression analysis for explaining as much variation in the dependent variable as can be explained with a fixed number of factors. PCA associates the first factor with the eigenvalue of the covariance matrix that points in the direction of maximum dispersion of returns. At each subsequent step, PCA then chooses the next factor to explain as much of the residual variation as can possibly be explained with an additional factor.

PCA is often called a *dimension reduction* technique since it encodes a lot of information with a parsimonious set of factors.

What does it mean that Quantal's factors are orthogonal?

Quantal's PCA methodology chooses successive factors to be perpendicular to the subspace of factors spanned by the previous ones. Hence, the eighth eigenvector points in the direction of maximum dispersion to the complementary subspace spanned by the first seven eigenvectors, for example. As a consequence, the covariance matrix of factor returns for a 30 factor model is I_{30} , the 30×30 identity matrix.

Explicit factor models have a non-trivial covariance matrix of factor returns, and this covariance matrix must be estimated in order to express the correlations among factor returns.

How does Quantal's model differ from explicit factor models?

Explicit factor models come in two varieties: macroeconomic factor models and fundamental factor models.

Macroeconomic models attempt to explain security returns based on unexpected changes to macro factors such as GDP or credit quality spread. The unexpected changes to the factor variables are observed, and the factor sensitivities are estimated via regression analysis.

Fundamental models, like Barra's, combine style factors and industry factors. Style factor sensitivities are standardized (z-score) values, and industry factor sensitivities are 0-1 values indicating whether a stock belongs to the specified industry. In contrast to macroeconomic models, the factor returns are estimated via regression analysis.

Macroeconomic models tend to use as few independent variables as possible to explain security returns (3 to 5 is typical). Fundamental models use as many independent variables as required to best explain the variation in security returns. Barra, for example, uses about 10 to 15 style factors and 50 to 60 industry factors.

In Quantal's statistical model, neither the factor sensitivities nor the factor returns can be observed; both are estimated in the risk estimate generation process by the model.

Which is better: explicit or implicit?

Implicit factor models tend to have better explanatory power and flexibility for mapping onto explicit factors. The shortcoming of implicit factor models lies in the abstract nature of its factors—they are mathematical constructs with no real-world interpretation.

How does Quantal's risk model avoid misspecification and multicollinearity?

Quantal's implicit factor model avoids misspecification since the factors are not identified with specific economic or fundamental variables. Hence, the Quantal risk model factors need not be pre-specified and have the flexibility to adapt to changing market conditions.

By virtue of being orthogonal, Quantal's implicit factors avoid multicollinearity by definition. As no linear combination of Quantal factors can replicate any other factor, the Quantal risk model avoids the

redundancy and overlap of factors inherent to explicit models.

Can Quantal recover explicit factors?

Yes. Quantal can take its 30 implicit factors and map them onto real-world explicit factors for country, industry, P/E, P/B, dividend yield, and other styles.

Given a matrix B_t of Quantal factor exposures and a second matrix H of explicit exposures, we can project via the regression

$$B_t = HQ_t + \Lambda$$

and solve for the desired mapping matrix

$$Q_t = (H' H)^{-1} H' B_t.$$

Once the projection matrix

$$\hat{B}_t = HQ_t$$

has been computed, the systematic risk can be expressed by

$$HGH' = \hat{B}_t \hat{B}_t'$$

and one can then solve for the matrix G giving the covariances of the explicit factors.

See also the graphic in Appendix B: Risk Attribution.

How does Quantal model fixed income instruments?

Quantal gets key rates for government issued bonds from major countries, interpolates the rate curves, and then uses the rate curves to price fictitious zero-coupon instruments at six month intervals. The cash flows from any fixed income instrument can be reproduced by a series of Quantal zero-coupon bonds. The risk of that bond is then calculated from the combination of the risks of the component zeros.

How does Quantal model derivatives?

Quantal uses its proprietary Custom ETF feature to replicate the risk characteristics of derivatives based on the risks of the underlying instruments. A Custom

ETF is simply basket of Quantal covered assets and some specified amount of cash.

For example, an at-the-money AAPL call with a delta of 0.5 is replicated by a Custom ETF containing 50 shares of AAPL stock and a short position in cash. Over time, as AAPL stock changes price and the contract expiration date approaches, Quantal dynamically adjusts the hedge ratio to faithfully replicate the call position and assess its risk.

How does Quantal compute a covariance matrix?

The output of the daily risk generation process is a *flat file* of B 's and D 's: 30 risk exposures (the B 's) and one residual (the D) for each of N securities in the coverage universe. Under the usual regression model assumptions that the residuals are uncorrelated with the factor returns and with each other, the daily conditional covariance matrix can be obtained by

$$V_t = BB' + \text{diag}(D),$$

where $\text{diag}(D)$ is the $N \times N$ diagonal matrix of residuals and represents idiosyncratic variance.

To annualize the daily covariance matrix, simply multiply by the number of trading days in a year:

$$V_t \rightarrow 252 \cdot V_t.$$

What does it mean that Quantal's covariance matrix is *conditional*?

In this context *conditional* means conditional on the security returns data up until the present time. Hence, the covariance matrix V_t indicates that the pair-wise asset correlations depend on the input returns data and risk estimates up until time t .

What is the half-life of the exponential weighting?

The Quantal risk model weights factor returns so that the most recent data contributes the greatest preponderance of information to the risk estimates. Factor returns incorporate a horizon of about 160 trading days of returns, and the exponential weight is calibrated for a half-life of 40 trading days.

Competitors' models typically use a half-life of 90 to 180 trading days depending on the short-term vs. long-term nature intent of the model. We deem the Quantal model to be more reactive to changing market conditions because of its shorter half-life.

How does Quantal compute portfolio risk and active risk?

Once the covariance matrix has been calculated, risk measures are computed by pre- and post-multiplication of the appropriate weight vector. Total daily portfolio variance for a portfolio with weight vector w is

$$\sigma_p^2 = w'(BB' + \text{diag}(D))w.$$

For a benchmark with weight vector b , active risk (aka, tracking error) is

$$\sigma_A^2 = (w - b)'(BB' + \text{diag}(D))(w - b).$$

To convert either of these to risk as measured by standard deviation, simply take the square root of the variance.

How does Quantal compute partial betas and factor portfolios?

See Appendix A: Partial Risk Exposures to Multiple Benchmarks and "Factor Portfolios".

How does Quantal compute risk attribution?

See Appendix B: Risk Attribution.

How are expected returns calculated?

The optimizer uses a quadratic utility function which rewards for expected return while penalizing for risk:

$$U = w' \Pi - \frac{\delta}{2} w' \Sigma w.$$

Here, w is a vector of portfolio weights, Π is a vector of expected returns, δ is the risk aversion parameter, and Σ is the Quantal provided covariance matrix.

The utility function can be modified to account for linear and quadratic transaction costs (to handle trading costs and market impact), different risk aversion parameters for systematic vs. idiosyncratic risk, and various other constraints.

The first-order solution to

$$\frac{\partial U}{\partial w} = 0$$

allows Quantal to calculate Black and Litterman *information neutral returns* according to

$$\Pi = \delta \Sigma w.$$

We calibrate the model to an average societal risk tolerance such that a market volatility of 20% corresponds to a market excess return of 8%.

Users can also optimize their portfolios using their own expected returns.

How does Quantal perform Monte Carlo simulation?

The Quantal return factors F_i are calibrated as normal random variates with mean 0 and standard deviation 1:

$$F_i \sim N(0,1).$$

As such, the return r_i and price p_i of security i can be simulated by

$$r_i = B_i \cdot z_i + D_i \cdot y_i$$
$$p_i \rightarrow (1 + r_i) \cdot p_i$$

where $z_i \sim N(0,1)$ ³⁰, $y_i \sim N(0,1)$, and B_i and D_i are the Quantal risk estimates for security i .

We calculate simulated prices for each portfolio security and for each security underlying derivatives, reprice the derivatives and the portfolio based on the simulated prices, and then repeat the process for several thousand trial paths.

Can Quantal perform scenario analysis?

Yes. Scenario Analyses can be conducted using the partial betas of the pricing inputs in a portfolio to the

set of factor portfolios. The shocks are applied to pricing inputs using the shock value and the partial beta.

For example, let's say the factor being shocked is the S&P 500 and we have two instruments in the portfolio, Goldman Sachs stock and a call option on the Russell 2000. The partial beta of Goldman Sachs (price = \$100) to the S&P 500 is 2.0 and the partial beta of the Russell 2000 (index value = 800) to the S&P 500 is 1.5. Let's say the shock to the S&P 500 is -5%. Under this scenario the shocked value of the Goldman Sachs stock is

$$100 + 2.0(-.05)(100) = 90.$$

Likewise the shocked value of the Russell 2000 is

$$800 + 1.5(-.05)(800) = 740.$$

The price of the Goldman Sachs stock in the shocked portfolio is \$90 dollars per share. The option on the Russell 2000 is revalued using an index value of 740.

Does Quantal have an asset or a credit model?

Quantal has an asset model that is consistent with the equity factor model. Quantal is also currently working to develop a Merton-type structural model based on credit default swap spreads.

References

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Grinold, Richard and Ron Kahn. *Active Portfolio Management*, Second Edition, McGraw-Hill, 1999.

Shlens, Jonathon. "A Tutorial on Principal Component Analysis", December 10, 2005. Available at www.sn1.salk.edu/~shlens/pub/notes/pca

Tien, David, and Paul Pflleiderer, Robert Maxim, and Terry Marsh, "Decomposing factor exposure for equity portfolios", *Linear Factor Models in Finance*, Chapter 13: Ed. by John Knight and Stephen Satchell, Elsevier Finance, Amsterdam.

Appendix A

Partial Risk Exposures to Multiple Benchmarks and “Factor Portfolios”

We represent a sampled covariance matrix by Σ . Define a series of weight vectors w_{B1}, \dots, w_{BM} representing weights of securities in the benchmark portfolios B_1, \dots, B_M , which we then concatenate to form the weight matrix W :

$$W_{N \times M} = \begin{bmatrix} w_{B1,1} & \dots & w_{BM,1} \\ \vdots & \ddots & \vdots \\ w_{B1,N} & \dots & w_{BM,N} \end{bmatrix} = [w_{B1} \ w_{B2} \ \dots \ w_{BM}]$$

N is the number of unique securities in the benchmarks, M is the number of benchmark portfolios, and each weight w_{Bi} is an $N \times 1$ column vector.

Now define the individual asset returns $Ra_{N \times T}$ where there are N assets over T time periods. If we multiply these by the weights in each of the B_1, \dots, B_M benchmarks we generate the benchmark returns over time. These can be concatenated into the return matrix R as shown below:

$$R_{T \times M} = \begin{bmatrix} r_{B1,1} & \dots & r_{BM,1} \\ \vdots & \ddots & \vdots \\ r_{B1,N} & \dots & r_{BM,N} \end{bmatrix} = [r_{B1} \ r_{B2} \ \dots \ r_{BM}]$$

where $r_{Bi} = (w'_{Bi} Ra)$ for $i = 1, \dots, m$.

If R and r_p have been demeaned, where $r_p = (w'_p Ra)$ is the returns of the portfolio over T periods and w_p is the weight vector of the N assets in the portfolio, then $R'R$ is simply the $M \times M$ covariance matrix between the M benchmarks, and $R'r_p$ is a vector of covariances between the M benchmarks and the portfolio. Recalling the classic equation for ordinary least squares regression gives

$$r_p = R\beta = \beta_{p,B1} r_{B1} + \beta_{p,B2} r_{B2} + \dots + \beta_{p,BM} r_{BM} + \epsilon$$

and

$$\hat{\beta} = (R'R)^{-1}(R'r_p) = (W'\Sigma W)^{-1}(W'\Sigma w_p)$$

which is a $M \times 1$ column vector.

We now substitute the Quantal forecasted covariance matrix $\hat{\Sigma}$ into the equation above:

$$\hat{\beta} = (W'\hat{\Sigma}W)^{-1}(W'\hat{\Sigma}w_p)$$

which gives us our final result. We can calculate this for each security in the universe and, hence, control the weighted partial exposure of the optimal portfolio to each of the benchmarks.

These betas can be used to control the exposure of the optimal portfolio to any factor, assuming a factor-mimicking portfolio can be devised. In addition, it should be noted that the partial betas are calculated conditional to the existence of any other portfolios included in the calculation. As a result, the partial beta calculated for two portfolios at once will not be the same as two total betas calculated one at a time.

Appendix B

Risk Attribution

The basic factor model for the returns R_t on N stocks for period t is:

$$R_t = B_t F_t + \epsilon_t. \quad (1)$$

Following Tien et al (2005), we attribute the $N \times K$ factor exposures B_t to a set of j characteristics of those stocks arranged in the columns of a $j \times K$ matrix Q_t :

$$B_t = H Q_t + \Lambda_t \quad (2)$$

where Λ_t are the components of systematic risk that are not accounted for in stocks that have only the characteristics j . This unexplained systematic risk is assumed orthogonal to the included characteristics. Note that even if the cross-sectional characteristics in (1) were sufficient to span (1), they may not do so if they are estimated on (say) long histories where the factors have shifted – that is, it is not sufficient that all the characteristics considered could “in principle” explain the factor risk in a portfolio.

$$R_t = (H Q_t + \Lambda_t) F_t + \epsilon_t. \quad (3)$$

The $N \times N$ conditional variance-covariance matrix V_t is then:

$$V_t = (H Q_t + \Lambda_t) F_t F_t' (H Q_t + \Lambda_t)' + D_t. \quad (4)$$

The projected portfolio variance σ_p^2 is, when the portfolio weights are in the $N \times 1$ vector w :

$$\sigma_p^2 = w' V_t w = w' (H Q_t Q_t' + \Phi_t) w + w' D_t w. \quad (5)$$

Notice that in a “factor” model framed in terms of the explicit characteristics, the “missed” systematic risk Φ_t will show up in the (non-diagonal) residual matrix, i.e.

$$\sigma_p^2 = w' V_t w = w' (H Q_t Q_t' H') w + w' (\Phi + D_t) w. \quad (6)$$

We can readily express this in terms of the risk attribution as in, say, Grinold and Kahn (2000, p. 83). Define a vector of exposures of the portfolio to the j characteristics as:

$$x_p' = w' H. \quad (7)$$

To relate to Grinold and Kahn, consider the benchmark tracking / enhanced index problem where the portfolio weights w are defined as active weights. Then Grinold and Kahn’s definition of FMCAR (“Factor Marginal Contribution to Active Risk”) is:

$$\frac{\partial \sigma_p^2}{\partial x_p} = \frac{QQ' x_p}{\sigma_p^2} \tag{8}$$

Each row of the $j \times 1$ vector in (8) is the portfolio's systematic risk that is attributed to the characteristic j of stocks (what Grinold and Kahn call "factor j "). In the case where the portfolio weights are used in (7) instead of the active weights, one can assume that there must be a concept analogous to FMCAR, labeled FMCR.

If we pre-multiply (8) by x_p we get the scalar $x_p' QQ' x_p / \sigma_p^2$, which is the percent of factor risk explained by all the characteristic risks in the set considered.

As a post-script, it seems that it would not be appropriate to derive this risk attribution by doing an unrestricted time series regression of the time series of factor returns F_t in (1) on the time series returns for portfolios formed on the basis of the characteristics (e.g. industry returns), since the coefficients in those regressions won't necessarily be consistent with the coefficients B_t .

The graphic below summarizes how Quantal's implicit factors are decomposed into explicit factors that may be used by investment managers.

Risk Decomposition

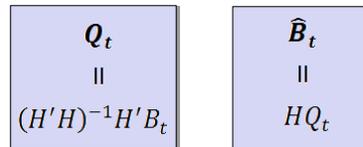
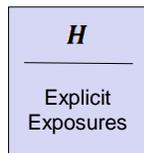
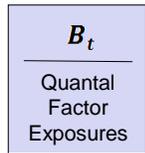
Goal: Map Quantal's **implicit** factors onto real-world **explicit** factors for country, sector, industry, P/E, P/B, dividend yield, and other style factors

Project: via the regression

$$B_t = HQ_t + \Lambda$$

To get:

Given: Matrices



Q_t is the desired mapping matrix!

Solve: systematic risk equation for G

$$HGH' = \hat{B}_t \hat{B}_t'$$

gives covariances of explicit factors