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Flight to Quality and Asset Allocation in a Financial Crisis

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With respect to the recent financial crisis, the authors argue that the appropriate adjustments to portfolio allocations in response to the market dislocation are determined by equilibrium considerations (supply must equal demand) and depend on individual investors' characteristics relative to societal averages. Using a simple model that captures the magnitude of the recent crisis, the authors show that the optimal tactical adjustments for most portfolios require a turnover of less than 10%.

Following the 2007–09 financial crisis, a sharp and sometimes acrimonious debate ensued about the usefulness of financial models in the throes of that crisis, including their value in making asset allocation decisions. Few would dispute the basic stylized facts that describe the recent crisis: As asset prices dropped sharply, the volatility of asset returns increased substantially; correlations between returns both within and across asset classes increased substantially as market factors increased in relative importance.[1] And, as recent survey evidence suggests, average risk tolerance decreased as wealth decreased and risk concerns became more salient. As the crisis unfolded, a big question for most investors was what tactical adjustments they should make to their portfolio holdings given the large losses in wealth they had suffered and the extreme market conditions they faced.

In our study, we addressed this question by considering how the simple economics of supply and demand play out in a situation where both risk and risk aversion have increased significantly. We developed a very simple model calibrated to capture the stylized facts that describe the recent crisis, and within that model, we considered how investors should trade among themselves as conditions change. Our goal was not to explain in a precise way what happened in the crisis of 2008; no simple model is capable of doing that. Rather, we wanted to illustrate how important it is to account for the basic laws of supply and demand in determining one's tactical response to a major market disruption of the magnitude we witnessed in 2008.

Perhaps the most natural response to a crisis, at least for many investors, is to "flee to safety" as confidence in the market erodes and prospects appear to dim, especially for equities. Of course, any investor who flees to safety must convince another investor to take the other side of his trade and "flee" to increased risk. Thus, only a subset of investors can flee to safety. We based our analysis on this simple observation, an observation that should be obvious but is unfortunately quite often forgotten (or ignored) in discussions of asset allocation and tactical responses to changed market conditions. In a crisis, asset prices must adjust so that a substantial number of investors find it in their interest to hold risky assets despite the increased uncertainty in the economy. Indeed, market clearing essentially requires that the "average" investor be willing to hold the available assets, including risky assets, in roughly their market proportions—which does not mean that no investor will want to make adjustments when market conditions change. Many will. But for someone who wants to make a change in one direction, there must be someone else willing to make the same change in the other direction.

The direction and size of the optimal adjustment for any particular investor depend on individual circumstances. One of the most important is the investor's tolerance for risk relative to that of the average investor.[2] In the simple examples that we used in our study, investors who have a lower tolerance for risk than the average investor (i.e., a higher ratio of fear to greed) want to adjust their portfolios to be more conservative when risk increases, whereas those who are more risk tolerant than average want to make adjustments that

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expose them to even more risk but allow them to earn commensurately higher prospective returns. In other words, the fall in risky asset prices in the crisis is sufficient to create a trade-off between risk and return that motivates the more risk-tolerant investors in the economy to take on more risk, accommodating those investors who want to reduce their risk exposure.

As mentioned earlier, our simple model depicts a financial crisis as severe as the 2007–09 crisis. Using this model in our study, we assumed that once our crisis occurs, both asset volatilities and correlations increase substantially. These increases capture the higher uncertainty and the increased importance of common risk factors in a crisis. We also assumed that risk tolerances decline across investors. In our base case, a key consideration for how an investor should respond to these changes is the relation between that investor's risk tolerance and the average investor's risk tolerance. This average should be thought of in terms of a wealth-weighted average, not a simple average. Because the average is wealth weighted, a financial crisis like the recent one leads to a change in society's average risk tolerance even if no individual investor becomes less risk tolerant. The reason is that more risk-tolerant investors will generally have more risky holdings going into a crisis and will thus suffer greater decreases in wealth than will more conservative investors. The distribution of wealth will shift toward the less risk tolerant. This rearrangement of wealth causes an endogenous drop in the average investor's risk tolerance beyond that arising from any changes in individual investors' risk appetites. The opposite tends to occur in bull markets when risky assets appreciate in value.

■ *Discussion of findings.* We found that risk premiums on both equities and bonds increase substantially in response to the crisis conditions that we assumed. It is generally accepted that risk was greatly magnified in the recent crisis. Volatilities on all asset classes increased dramatically, and correlations between asset classes also increased, especially the correlations between equity and bond returns, which shifted from negative in the pre-crisis period to positive in the middle of the crisis. In our study, we attempted to capture this reality roughly in our calibrated model. In the crisis scenarios we considered, we found that equity premiums increase by 25%–35% and bond risk premiums generally increase by 7%. Increases of this magnitude are required to clear the market given the large increase in systematic risk for investors. Despite these large changes in risk premiums, our example shows that the magnitude of the rebalancing appropriate for most investors is not large, contrary to what many might believe. Total (economy-wide) turnover rates are about 5% for the base case we considered. Not surprisingly, we found that turnover is even less if leverage—short selling and margining—is restricted or risk tolerance remains constant.[3]

In our base case, we assumed that investors have homogeneous expectations. Of course, it is possible—some might argue obvious—that sharp differences in investor expectations could arise in the throes of a crisis. These differences could be due to information asymmetries, differences of opinion, or "animal spirits." Whatever the source of these differences, the laws of supply and demand still apply. To explore the effects of heterogeneous expectations and to contrast them with the effects of differences in risk tolerance, we looked at a case where all investors have the same risk tolerance but vary in their degree of optimism or pessimism. Specifically, we assumed that the most optimistic 5% of investors expect equities to receive returns 9% above the market consensus and bonds to receive returns 6% above the consensus, whereas the most pessimistic have the opposite forecasts. Perhaps not surprisingly, we found that the turnover generated by these assumed divergences in expectations is much greater than the turnover in our model with assumed divergent levels of risk tolerance.

We also considered what happens in the realistic case where a subset of investors follows a naive "target weight" rebalancing policy, which involves purchasing those assets that have fallen the most in price and selling those that have appreciated or fallen less than average. In other words, investors with a target weight policy are mechanically following a contrarian strategy and thus increase the demand for risky assets in a crisis relative to our base case. This outcome causes the increases in equilibrium risk premiums to be lower than they would otherwise be but involves greater turnover than in our base case. As a final variation on our base case, we considered the impact of greater heterogeneity in investor risk preferences. In particular, we assumed a uniform wealth distribution across all risk tolerance clienteles, rather than a distribution in which most investors are close to the average risk tolerance level. Risky asset returns and Sharpe ratios are higher in this case than in the base case: The higher demand for safe portfolios from the increased number of risk-averse investors is not quite offset by the risk-bearing capacity of the increased number of risk-tolerant investors. In this variation, the amount of turnover due to tactical responses to the crisis is greater than in our base case and is driven by the fact that greater heterogeneity in risk tolerance levels means that more trading is needed to distribute risk optimally.

Asset prices are determined in part by *ex ante* risk premiums, which can be expected to increase in a crisis owing to the factors described earlier. Our example illustrates how these risk premiums are jointly determined by the risk structure of returns, changes in risk tolerance levels, the distribution of wealth across those making allocation decisions, and such institutional features as margin rules and leverage constraints. In the 2007–09 crisis, the changes in risk premiums, together with the likely increased influence of macrodrivers on corporate profitability in a crisis, tended to increase the relative importance of marketwide fundamentals in asset price variation. When asset prices fall across the board in a crisis, *historically* measured risk premiums *decrease* if these historical averages include returns realized in the crisis, as is typically the case. Thus, historically measured risk premiums go down at a time when we know that forward-looking (*ex ante*) risk premiums are much increased. This result means that although it is *almost* useless to use a short history of past returns to estimate future returns in "normal times," it is *completely* useless to do so in a crisis.

Asset Allocation in a Financial Crisis: Base Case

To keep our analysis simple, we considered an example with only five asset classes: U.S. equities, (non-U.S.) developed-market equities, emerging-market equities, bonds, and cash (including such instruments as short-term U.S. Treasuries and other instruments that are essentially risk free). **Table 1** shows the assumptions we made about the market weights and the perceived distribution of the returns of the five asset classes before the arrival of our illustrative crisis.

As can be seen in Table 1, our example is calibrated so that before our crisis hits, riskless cash instruments represent 10% of the market, bonds represent 30% of the market, and the remaining slice of the market is divided roughly equally among equities in the United States, developed foreign markets, and emerging markets. We assumed that the supplies of these asset classes remain fixed—though not their market values. Under our assumptions, the volatilities of the risky asset returns in the period before our crisis range from a low of 4% for bonds to a high of 20% for emerging-market equities. These assumed volatilities are close to the realized volatilities on these asset classes in the period preceding the recent financial crisis—that is, from the beginning of 2000 until the Lehman Brothers bankruptcy.[4] We also assumed that the correlations among the equity asset classes in our pre-crisis period are all positive but not particularly high. Correlations of equity returns with bond returns are assumed to be slightly negative. These correlations are roughly in accord with the actual correlations that were realized in the pre-crisis period.[5] Our illustrative examples are meant only to be consistent in broad terms with what happened in the recent crisis. We are not arguing that any of these parameters precisely captures what investors were expecting in the pre-crisis period. We do believe, however, that the parameters in Table 1 roughly represent the structure of the market before the financial crisis of 2007–2009.

As Table 1 shows, we assumed a pre-crisis rate of return of 3% for riskless investments, which we considered set largely by U.S. Federal Reserve Board policies. We also assumed that investors who want to leverage their portfolios in the pre-crisis period can borrow at a 50 bp spread over Treasuries. The last column of Table 1 shows the equilibrium expected returns for the various asset classes based on our assumptions; these equilibrium expected returns can be thought of as the required returns that equate supply with demand. We know that market prices must be set so that

Table 1. Assumed Pre-Crisis Market Conditions

			Correlations				
	Market Weight	Standard Deviation	U.S. Equities	Dev. Equities	Em. Equities	Bonds	Equilibrium Expected Return[a]
U.S. equities	20%	18%	1.00	0.50	0.35	–0.15	5.36%
Dev. equities	22	16	0.50	1.00	0.50	–0.05	5.30
Em. equities	18	20	0.35	0.50	1.00	–0.05	5.66
Bonds	30	4	–0.15	–0.05	–0.05	1.00	3.06
Cash	10	0					3.00[b]

[a]Average risk tolerance = 0.5.
[b]Borrowing cost = 3.50%.

investors are (in aggregate) willing to hold in their portfolios all the assets available in the market.[6] If prices in an asset class are too high, expected returns on those assets will be too low and demand for the assets will be insufficient. Conversely, if prices of a set of assets are too low, perceived expected returns will be too high and investors will want to hold more of those assets than is available (i.e., there will be excess demand). To determine these equilibrium expected returns, we made some assumptions about investors' risk tolerance because risk premiums cannot be determined unless risk tolerance levels are specified. Because one of the goals of our study was to determine how investors should trade and adjust their portfolios in response to a shock, in our base case we explicitly accounted for the fact that not all investors are the same and that, in particular, they differ in their risk tolerance.[7] For the base case, we assumed that before our crisis hits, the average investor has a risk tolerance of 0.5, albeit with a cross-sectional distribution.[8] Our assumptions about the distribution of investors by risk tolerance are shown in the first few rows of **Table 2**.

As shown in Table 2, we assumed seven different "clienteles" of investors, whereby clienteles are distinguished by risk tolerance. The largest clientele comprises investors with risk tolerance equal to 0.5, the societal average. This group is assumed to control 30% of the total wealth portfolio. Clienteles with risk tolerances of 0.4 and 0.6 each control 20% of the total wealth. The least risk-tolerant clientele (whose risk tolerance equals 0.2) has only 5% of the wealth, which matches the wealth of the most risk-tolerant clientele (whose risk tolerance equals 0.8).[9] Let us assume for the moment that investors, on average, have the return expectations given in the last column of Table 1, what we are calling equilibrium expected returns. This assumption means, for example, that investors, on average, are forecasting that the U.S. equity market will return a little over 5% and emerging markets will return a bit more than 5.5%. Investors in each clientele will want to adjust their portfolios to obtain the best trade-off between risk and return given their preferences, which entails identifying the efficient frontier (i.e., the set of allocations that gives the highest expected returns for various risk levels) and then choosing from among these efficient allocations the one that gives the preferred trade-off between risk and return. Given our assumptions, including the posited expected returns, we can see that investors in clientele 4 (i.e., the average investors) hold about 61% of their wealth in equities, 39% in bonds, and a trivial amount in cash. The most risk averse have substantial holdings in cash and bonds with little in equities, whereas the most risk tolerant invest heavily in equities and have no holdings in cash.

Table 2 shows that our posited equilibrium required returns are indeed consistent with supply equaling demand. Panel B of Table 2 shows each clientele's holdings in the four risky asset classes and in cash as a percentage of total market wealth. For example, let us consider clientele 3. Investors in this clientele allocate 14.58% of their wealth to emerging markets. Because investors in this clientele represent 20% of total wealth, they

Table 2. Pre-Crisis Allocations for Seven Investor Clienteles Differing by Risk Tolerance

Clientele	1	2	3	4	5	6	7	Total
Risk tolerance	0.2	0.3	0.4	0.5	0.6	0.7	0.8	
% of total wealth	5.00	10.00	20.00	30.00	20.00	10.00	5.00	
A. Optimal allocations								
U.S. equities	8.31%	12.47%	16.63%	20.56%	23.55%	26.55%	29.54%	
Dev. equities	8.93	13.39	17.86	22.23	26.21	30.19	34.17	
Em. equities	7.29	10.94	14.58	18.16	21.47	24.77	28.07	
Bonds	17.10	25.65	34.20	39.05	28.77	18.49	8.21	
Cash	58.36	37.54	16.72	0.00	0.00	0.00	0.00	
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	
B. % holdings in economy								
U.S. equities	0.42%	1.25%	3.33%	6.17%	4.71%	2.65%	1.48%	20.00%
Dev. equities	0.45	1.34	3.57	6.67	5.24	3.02	1.71	22.00
Em. equities	0.36	1.09	2.92	5.45	4.29	2.48	1.40	18.00
Bonds	0.86	2.57	6.84	11.72	5.75	1.85	0.41	30.00
Cash	2.92	3.75	3.34	0.00	0.00	0.00	0.00	10.00
Total	5.00%	10.00%	20.00%	30.00%	20.00%	10.00%	5.00%	100.00%

end up allocating 2.92% (i.e., 20% × 14.58%) of total wealth to emerging markets. If we add up all the allocations made by each clientele to emerging markets (0.36% + 1.09% + 2.92% + 5.45% + 4.29% + 2.48% + 1.40%), we find a total allocation to emerging markets across all investors of 18%, which is precisely our assumed weight of emerging markets in the total wealth portfolio. In other words, demand for emerging-market equities is equal to supply, as it must be if the market is in equilibrium. This equality between supply and demand holds for all four risky asset classes (and for cash). The posited expected returns on the four risky asset classes are, therefore, consistent with the equilibrium in our pre-crisis scenario. One aspect of the equilibrium expected returns in our example that might initially be puzzling is the very low risk premium on bonds; the expected return on bonds (3.06%) is only marginally higher than the assumed return on cash (3%). This aspect becomes understandable once we note that bonds are assumed to have the negative correlations with the equity asset classes that they did in fact have over the pre-crisis period (from the beginning of 2000 to the Lehman Brothers bankruptcy). If investors forecasted positive, rather than negative, correlations between bonds and equities, the risk premium on bonds would be higher in our pre-crisis scenario.

We assumed that once our crisis hits, the market value of most risky assets plunges and risk increases dramatically. For purposes of illustration, let us assume that equities fall 40% in value across the board, bonds fall 10%, and nominally riskless assets increase 5%. We argue not that this is a precise representation of what happened over any specific period in the recent crisis but, rather, that it roughly corresponds to the general level of losses and gains in the last part of 2008. As mentioned earlier, another likely consequence of the events of 2008 was an overall decrease in risk tolerance.[10] To make our scenario consistent with this possibility, we assumed that with the onset of our crisis, the risk tolerance of each clientele decreases uniformly by 0.10. This assumption means, for example, that the average investor's risk tolerance decreases from 0.5 to 0.4.[11]

The effects of the changes in asset values are shown in **Table 3**. Because bond prices fall by only 10% and equity prices fall by 40%, bonds necessarily become a more important part of the total market after the crisis. They represent almost 37%, rather than 30%, of the total wealth portfolio. U.S. equities, which are 20% of the total wealth portfolio before our crisis, are only 16.33% after our crisis hits.

As mentioned earlier, there is a less obvious effect of the changes in asset values: The less risk-tolerant clienteles become slightly more important (in terms of the wealth they control) and the more risk-tolerant clienteles become less important. Clientele 1 is assumed to control 5% of total wealth before the crisis but controls 6.22% after the crisis. The increase is the consequence of the fact that this risk-averse clientele allocates less to equities before the crisis and thus suffers a lower

Table 3. Post-Crisis Investor Holdings before Allocation Adjustments

Clientele	1	2	3	4	5	6	7	Total
New level of risk tolerance	0.1	0.2	0.3	0.4	0.5	0.6	0.7	
New % of total wealth	6.22	11.51	21.17	29.27	18.67	8.92	4.25	
A. Allocations after 40% decline in equities, 10% decline in bonds, and 5% gain in riskless assets								
U.S. equities	5.46%	8.85%	12.83%	17.20%	20.59%	24.30%	28.38%	
Dev. equities	5.86	9.50	13.77	18.60	22.91	27.63	32.82	
Em. equities	4.79	7.76	11.25	15.20	18.77	22.67	26.96	
Bonds	16.84	27.29	39.57	49.01	37.73	25.39	11.84	
Cash	67.05	46.60	22.58	0.00	0.00	0.00	0.00	
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	
B. % holdings after 40% decline in equities, 10% decline in bonds, and 5% gain in riskless assets								
U.S. equities	0.34%	1.02%	2.71%	5.03%	3.85%	2.17%	1.21%	16.33%
Dev. equities	0.36	1.09	2.92	5.44	4.28	2.46	1.39	17.96
Em. equities	0.30	0.89	2.38	4.45	3.50	2.02	1.15	14.69
Bonds	1.05	3.14	8.38	14.34	7.05	2.26	0.50	36.73
Cash	4.17	5.36	4.78	0.00	0.00	0.00	0.00	14.29
Total	6.22%	11.51%	21.17%	29.27%	18.67%	8.92%	4.25%	100.00%

loss in value than the much more risk-tolerant clienteles. This outcome means that the average risk tolerance declines even more than is suggested by the uniform 0.10 reduction in risk tolerance that we assumed for each clientele. Because the clienteles with low risk tolerance become wealthier than the clienteles with high risk tolerance, the average risk tolerance weighted by wealth level declines from 0.5 to 0.386.

Table 4 reports our assumptions concerning the change in the risk structure brought about by our illustrative crisis. As Table 4 shows, we assumed that the volatility of each asset class significantly increases as a consequence of our crisis and the greater uncertainty created by an economic dislocation.[12] In addition, we assumed that correlations among asset classes rise because of the increased importance of marketwide factors and that the sign of the correlation between equities and bonds switches to positive in the crisis.[13]

Given all the changes brought about by our assumed crisis conditions, what is the appropriate tactical response an investor should make in terms of his portfolio allocation? Because risk increases substantially and investors' risk tolerance decreases, it might seem that investors should uniformly move to more conservative portfolio allocations and "wait out the storm." Indeed, this advice was given to many investors in the 2007–09 crisis, and no doubt there was a "flight to safety" phenomenon at the time. But as we observed earlier, not everyone can retreat to safety. Some investors must hold the risky assets. The laws of supply and demand still apply, even in a crisis. Prices of risky assets must decrease and expected returns must increase enough for investors, in aggregate, to be willing to hold all the assets available. The last column of Panel B in Table 4 shows, for each asset class, the new level of (annualized) equilibrium expected return that equates supply with demand. We based these equilibrium required rates on the assumption that short-term riskless rates have been lowered to 1% through central bank (i.e., Fed) policies that would be implemented in response to our crisis. Recall that we assumed that in the period before our crisis, investors pay only a 50 bp spread above Treasuries for borrowing. This spread would quite likely increase rather dramatically in a crisis of the sort we are assuming. This scenario no doubt occurred in 2008 given, among other things, the problems in the credit markets that followed the Lehman Brothers bankruptcy. Thus, we assumed that investors need to pay a 3% rate to borrow after our crisis sets in, representing an increase in the spread from 50 bps to 200 bps. Again, we emphasize that we are not attempting to capture precisely the effects of the 2007–09 crisis, but we do believe that this estimate could be considered reasonable for the increase in the spread for borrowing that occurred during that time.

Given all our assumptions, we can see that equity risk premiums after our crisis starts must be 30%–35% higher than their pre-crisis values, and the bond risk premium must be about 7.5% higher. These dramatic increases in risk premiums are due to increased volatilities, decreased levels of risk tolerance, and a decreased ability to

Table 4. Pre- and Post-Crisis Market Conditions

	Market Weight	Standard Deviation	Correlations: U.S. Equities	Correlations: Dev. Equities	Correlations: Em. Equities	Correlations: Bonds	Equilibrium Expected Return[a]
A. Pre-crisis							
U.S. equities	20.00%	18.00%	1.00	0.50	0.35	–0.15	5.36%
Dev. equities	22.00	16.00	0.50	1.00	0.50	–0.05	5.30
Em. equities	18.00	20.00	0.35	0.50	1.00	–0.05	5.66
Bonds	30.00	4.00	–0.15	–0.05	–0.05	1.00	3.06
Cash	10.00	0.00					3.00[b]
B. Post-crisis							
U.S. equities	16.33%	62.00%	1.00	0.45	0.50	0.10	33.20%
Dev. equities	17.96	54.00	0.45	1.00	0.85	0.45	36.37
Em. equities	14.69	60.00	0.50	0.85	1.00	0.30	39.25
Bonds	36.73	18.00	0.10	0.45	0.30	1.00	8.59
Cash	14.29	0.00					1.00[c]

[a]Average pre-crisis risk tolerance = 0.5. Average post-crisis risk tolerance = 0.386.
[b]Borrowing cost = 3.50%.
[c]Borrowing cost = 3.00%.

reduce risk through diversification (higher correlations). Risk premiums in excess of 30% may at first seem unreasonably—perhaps absurdly—high. It is important to understand that these risk premiums should be interpreted not as the rate of return investors would expect over a full year but, rather, as the (annualized) short-term premiums investors would expect during that part of a crisis when uncertainties are most pronounced, which may be quite a bit less than a year.[14]

In interpreting our results, one might well ask how important are the increases in correlations and the accompanying loss of diversification opportunities relative to the overall increase in volatilities. **Table 5** gives some insight into the relative effects.

The first column shows the change in risk premiums that would occur if the volatilities of equities increase as we assumed (e.g., we assumed that our crisis causes the volatility of U.S. equities to increase from 18% to 62%) but there is *no* change in the correlation structure (e.g., bonds remain negatively correlated with equities). There is still a dramatic increase in equity risk premiums and a significant increase in the risk premium on bonds. The second column reports the results for the opposite assumption—namely, that the asset class volatilities do not increase but the correlations among asset classes do. Here, we can see a much smaller effect on risk premiums. One might be tempted to conclude that the increased risk premiums in our crisis scenario are almost wholly due to the increases in asset class volatilities and that the correlation structure is unimportant. This conclusion, however, is incorrect. The third column reports the actual total increase in risk premiums. We can easily see that the total effect is not equal to a sum of the independently measured volatility effects and the correlation effects. In fact, once there has been a substantial increase in volatilities, the incremental effect of an increase in correlations is quite significant, as seen in the last column. The incremental effect of the increased correlations for the equity risk premiums ranges from 4% to 11%, and the effect for bonds is almost 5%. There is clearly an interaction between the two effects. When volatilities are high, the effect on risk premiums of increases in correlations is much higher than when volatilities are low.

At this point, we need to be very clear about what is behind the calculation of these risk premiums. We are assuming as a given the changes in valuations that occur in our crisis (i.e., the fall of 40% in equity values and 10% in bond values). In a crisis, these changes in valuations would be due in part to downward revisions in investors' expectations about the future cash flows that assets will deliver—stocks would be forecasted to pay lower dividends in the future because of a bleak economic outlook. The downward valuations would also be due to increases in the rate of discount applied to the expected future cash flows because of increased risk premiums. The increased pessimism about future earnings is related to adjustments in the numerator of a discounted cash flow analysis, and the increases in the rate of discount are related to adjustments in the denominator. Our assumptions about how the market risk structure (volatilities and correlations) and clientele risk tolerances change allow us to infer how risk premiums would change, which determines how big the "denominator effect" would be. From this, one can then infer what the "numerator effect" would be. If we change assumptions about risk structure and risk tolerance, doing so will change what would be attributed to the numerator and denominator effects. Those assets whose volatilities and correlations with other assets increase the most in a crisis—and that thus have the largest crisis-induced increases in premiums—would, via the denominator effect, have the biggest declines in price. It is also reasonable to expect that the assets whose volatilities and correlations are worst affected by a crisis would have higher *pre-crisis* cross-sectional premiums and lower prices, all else being equal.[15]

We next considered the trading that occurs in our crisis scenario as investors adjust to the new distribution of returns. Panel A of **Table 6** shows

Table 5. Change in Risk Premium Due to Increases in Volatility and Correlations

	Change in Risk Premium			
	Volatility Increases/ Correlations Unchanged	Volatility Unchanged/ Correlations Increase	Total	Incremental Effect of Increase in Correlations
U.S. equities	25.82%	0.43%	29.84%	4.02%
Dev. equities	24.81	0.81	33.07	8.26
Em. equities	24.33	1.18	35.58	11.26
Bonds	2.65	0.41	7.53	4.88

Table 6. Post-Crisis Investor Holdings after Allocation Adjustments

Clientele	1	2	3	4	5	6	7	Total
Risk tolerance	0.1	0.2	0.3	0.4	0.5	0.6	0.7	
% of total wealth	6.22	11.51	21.17	29.27	18.67	8.92	4.25	
A. New optimal allocations								
U.S. equities	4.43%	8.85%	13.28%	17.70%	20.37%	23.54%	27.47%	
Dev. equities	4.24	8.47	12.71	16.95	25.07	32.05	37.39	
Em. equities	4.00	8.00	12.00	16.00	18.20	20.92	24.41	
Bonds	12.18	24.35	36.53	48.70	36.37	31.18	36.38	
Cash	75.16	50.32	25.48	0.64	0.00	–7.70	–25.65	
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	
B. New % holdings in economy								
U.S. equities	0.28%	1.02%	2.81%	5.18%	3.80%	2.10%	1.17%	16.33%
Dev. equities	0.26	0.98	2.69	4.96	4.68	2.86	1.59	17.96
Em. equities	0.25	0.92	2.54	4.68	3.40	1.87	1.04	14.69
Bonds	0.76	2.80	7.73	14.26	6.79	2.78	1.55	36.73
Cash	4.67	5.79	5.39	0.19	0.00	–0.69	–1.09	14.29
Total	6.22%	11.51%	21.17%	29.27%	18.67%	8.92%	4.25%	100.00%
C. Change in allocations								
U.S. equities	–1.03%	0.01%	0.45%	0.51%	–0.22%	–0.76%	–0.91%	
Dev. equities	–1.63	–1.03	–1.07	–1.65	2.15	4.42	4.57	
Em. equities	–0.79	0.24	0.75	0.81	–0.57	–1.75	–2.55	
Bonds	–4.67	–2.94	–3.05	–0.31	–1.36	5.79	24.54	
Cash	8.11	3.72	2.91	0.64	0.00	–7.70	–25.65	
Total	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	
D. Change in % holdings in economy								
U.S. equities	–0.06%	0.00%	0.10%	0.15%	–0.04%	–0.07%	–0.04%	0.00%
Dev. equities	–0.10	–0.12	–0.23	–0.48	0.40	0.39	0.19	0.00
Em. equities	–0.05	0.03	0.16	0.24	–0.11	–0.16	–0.11	0.00
Bonds	–0.29	–0.34	–0.64	–0.09	–0.25	0.52	1.04	0.00
Cash	0.50	0.43	0.62	0.19	0.00	–0.69	–1.09	0.00
Total	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

the optimal allocations for different levels of risk tolerance for the posited new expected returns and the new risk structure. Panel B shows the percentage holdings that each clientele will have once they establish their optimal allocations. Investors in clientele 1, who now control 6.22% of the total wealth, wish to allocate 75.16% of that wealth to cash. Thus, after they make the trades necessary to establish this allocation, they will hold 4.67% (6.22% × 75.16%) of the total market wealth in the form of cash. As shown in Panel B of Table 6, the total allocation to cash across all clienteles is 14.29%, which is precisely the assumed market weight of cash in the total economy. Thus, supply of cash equals demand for cash. This outcome is true of the other asset classes as well, which shows that the posited equilibrium expected returns in Panel B of Table 4 do indeed equate supply with demand.

At the opposite end of the spectrum to least risk-tolerant clientele 1 is most risk-tolerant clientele 7. Investors in this clientele now want to take a rather aggressively leveraged position by borrowing slightly more than 25% of their wealth level. For this clientele, the sharply increased risk premiums on equities and bonds create a very attractive opportunity to increase expected returns at what they view, given their high risk tolerance, as a reasonable cost in increased volatility. Note that they are willing to do this even though we have assumed a rather substantial borrowing cost of 200 bps.

One key observation that can be drawn from Table 6 is that despite the pronounced changes in market conditions and risk premiums, the total amount of trading done in response to our crisis (i.e., the trading that achieves optimal risk sharing given the changed conditions) is somewhat limited. The total amount of turnover across all investors is only 4.95%.[16] A good part of this trading is attributable to the willingness of the most risk-tolerant investors (mainly those in clienteles 6 and 7) to take leveraged portfolio positions and, in doing so, allowing those who are the least risk tolerant to load up on cash or engage in a flight to quality. The clienteles that are close to the average in terms of risk tolerance do very little trading. The turnover percentages for clienteles 3, 4, and 5 are 4.11%, 1.96%, and 2.15%.[17]

One possibly puzzling result found in Table 6 is that the most risk-tolerant investors (clienteles 6 and 7) substantially *increase* their holdings of bonds. Investors in clientele 6 allocate 5.79% more of their wealth to bonds, and those in clientele 7 allocate 24.54% more. These substantial purchases of bonds might seem to go against the notion that these two clienteles should be taking on more risk. **Table 7** shows why this is happening. It reports the optimal portfolio of *risky* assets—the portfolio that includes equities and bonds but excludes cash—that each clientele should hold. As shown in Panel A of Table 7, before our crisis all clienteles with risk tolerances under 0.5 (clienteles 1–3) hold the *risky* assets in the same proportions. Their asset allocations differ only in the amount allocated to the risky asset portfolio and the amount allocated to cash. The more risk-tolerant clienteles (clienteles 4–7) hold the risky assets in slightly different proportions as they move out along the efficient frontier of risky assets. None of the more risk-tolerant clienteles choose to enhance returns through leverage. This result arises because we assumed that investors who want to obtain higher returns through leverage must borrow at a 50 bp spread over the return on cash, and given the equilibrium risk–reward structure before our crisis, they choose not to do so.

Now, let us consider the trading that the most risk-tolerant clienteles (6 and 7) will do after the crisis in order to move to their optimal allocations. First, note that given the changes in equilibrium risk premiums brought about by our crisis, these two clienteles choose to use leverage. Investors in the most risk-tolerant clientele wish to increase leverage (risky investment divided by total investment) from 100% to 126%, and those in the next most risk-tolerant clientele wish to increase leverage from 100% to 108%. Second, note that the optimal portfolio of risky assets to be leveraged is relatively heavily weighted toward bonds. This substantial

Table 7. Investor Holdings of Risky Asset Classes

Clientele	1	2	3	4	5	6	7
Risk tolerance	0.2	0.3	0.4	0.5	0.6	0.7	0.8
% of total wealth	5.00	10.00	20.00	30.00	20.00	10.00	5.00
A. Optimal risky asset portfolio before crisis							
U.S. equities	19.97%	19.97%	19.97%	20.56%	23.55%	26.55%	29.54%
Dev. equities	21.45	21.45	21.45	22.23	26.21	30.19	34.17
Em. equities	17.51	17.51	17.51	18.16	21.47	24.77	28.07
Bonds	41.07	41.07	41.07	39.05	28.77	18.49	8.21
Optimal leverage before (risky/total)	41.64%	62.46%	83.28%	100.00%	100.00%	100.00%	100.00%
Risk tolerance	0.1	0.2	0.3	0.4	0.5	0.6	0.7
% of total wealth	6.22%	11.51%	21.17%	29.27%	18.67%	8.92%	4.25%
B. Risky asset portfolio after crisis but before adjustments							
U.S. equities	16.57%	16.57%	16.57%	17.20%	20.59%	24.30%	28.38%
Dev. equities	17.79	17.79	17.79	18.60	22.91	27.63	32.82
Em. equities	14.53	14.53	14.53	15.20	18.77	22.67	26.96
Bonds	51.11	51.11	51.11	49.01	37.73	25.39	11.84
Leverage after crisis (risky/total)	32.95%	53.40%	77.42%	100.00%	100.00%	100.00%	100.00%
C. New optimal risky asset portfolio							
U.S. equities	17.82%	17.82%	17.82%	17.82%	20.37%	21.86%	21.86%
Dev. equities	17.06	17.06	17.06	17.06	25.07	29.76	29.76
Em. equities	16.11	16.11	16.11	16.11	18.20	19.43	19.43
Bonds	49.02	49.02	49.02	49.02	36.37	28.95	28.95
Optimal leverage after (risky/total)	24.84%	49.68%	74.52%	99.36%	100.00%	107.70%	125.65%

weight is due to the fact that the overall weight of bonds in the market has increased after our crisis and bonds must be priced to make them attractive to hold in larger proportions. The optimal weight of bonds in the risky asset portfolio is 28.95%. Because this amount is larger than what the two most risk-tolerant clienteles hold in bonds after our crisis but before they make adjustments (25.39% and 11.84%), each of these clienteles will be purchasing bonds in order to obtain the best risky asset portfolio to hold and lever up through borrowing.

Asset Allocation in a Financial Crisis: Alternative Crisis Scenarios

We next considered some significant variations in our base-case assumptions for additional insights that our simple framework can offer into the important issues concerning allocations and tactical responses in a crisis.[18] In our first variation, we assumed that investors cannot short-sell assets and, in particular, cannot borrow to achieve leverage. Intuitively, this constraint will limit the ability of the most risk-tolerant investors to take on more risk in the crisis relative to what they can do in our base-case scenario, in which leverage is possible (Table 7). **Table 8** shows the optimal allocations before and after the crisis when leverage is assumed to be infeasible or prohibitively expensive.

As can be seen in Table 8, trading without short selling is more limited and tactical responses are less pronounced. In fact, total aggregate turnover is only 3.23%, rather than the 4.95% in the base case. A comparison between the no-leverage case and the base case is shown in **Table 9**.

The results for the base case are given in the first column, and those for the no-leverage case are given in column A. Note that when leverage is infeasible, risk premiums are slightly higher after the crisis because investors with low risk tolerance must hold a slightly riskier position. Table 9 also gives the pre- and post-crisis Sharpe ratios. In the base case, the Sharpe ratio increases from 0.174 to 0.732 because of the crisis. When leverage is not allowed, the post-crisis Sharpe ratio is slightly higher than in the base case (0.747 versus 0.732). Overall, our example suggests that leverage constraints (margin policy, etc.) can be important in determining the degree to which a flight to quality is "absorbed" by the least risk-averse investors.

Table 8. Post-Crisis Investor Holdings before and after Allocation Adjustments with Leverage Restricted

Clientele	1	2	3	4	5	6	7
New level of risk tolerance	0.1	0.2	0.3	0.4	0.5	0.6	0.7
New % of total wealth	6.22	27.83	41.74	49.45	36.48	23.52	10.56
A. Allocations after 40% decline in equities, 10% decline in bonds, and 5% gain in riskless assets							
U.S. equities	5.46%	8.85%	12.83%	17.20%	20.59%	24.30%	28.38%
Dev. equities	5.86	9.50	13.77	18.60	22.91	27.63	32.82
Em. equities	4.79	7.76	11.25	15.20	18.77	22.67	26.96
Bonds	16.84	27.29	39.57	49.01	37.73	25.39	11.84
Cash	67.05	46.60	22.58	0.00	0.00	0.00	0.00
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
B. New optimal allocations							
U.S. equities	4.54%	9.08%	13.63%	17.72%	20.33%	22.94%	25.54%
Dev. equities	3.95	7.90	11.85	16.78	24.99	33.19	41.40
Em. equities	4.13	8.25	12.38	16.05	18.20	20.35	22.50
Bonds	13.91	27.83	41.74	49.45	36.48	23.52	10.56
Cash	73.47	46.93	20.40	0.00	0.00	0.00	0.00
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
C. Change in allocations							
U.S. equities	–0.92%	0.24%	0.80%	0.52%	–0.26%	–1.36%	–2.83%
Dev. equities	–1.91	–1.60	–1.93	–1.82	2.07	5.56	8.58
Em. equities	–0.66	0.50	1.13	0.85	–0.57	–2.33	–4.47
Bonds	–2.93	0.54	2.17	0.44	–1.25	–1.87	–1.28
Cash	6.42	0.33	–2.17	0.00	0.00	0.00	0.00
Total	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 9. Market Equilibrium for Optimally Adjusted Portfolios in the Base Case (A) and Four Alternative Scenarios (B–E)

	Base Case	No Leverage (A)	Equal-Wealth Clienteles (B)	No Decrease in Risk Tolerance (C)	Naive Rebalancing (D)	Differential Expectations (E)
A. Pre-crisis						
Average risk tolerance	0.500	0.500	0.500	0.500	0.500	0.500
U.S. equities	5.36%	5.36%	5.43%	5.36%	5.36%	5.32%
Dev. equities	5.30	5.30	5.37	5.30	5.30	5.26
Em. equities	5.66	5.66	5.73	5.66	5.66	5.62
Bonds	3.06	3.06	3.13	3.06	3.06	3.02
Cash	3.00	3.00	3.00	3.00	3.00	3.00
Sharpe ratio	0.174	0.174	0.181	0.174	0.174	0.170
B. Post-crisis						
Average risk tolerance	0.386	0.386	0.377	0.485	0.386	0.385
U.S. equities	33.20%	33.64%	34.50%	26.46%	29.94%	25.86%
Dev. equities	36.37	36.81	37.75	28.95	33.30	28.31
Em. equities	39.25	39.69	40.70	31.25	35.86	30.53
Bonds	8.59	9.08	9.30	6.93	7.22	6.63
Cash	1.00	1.00	1.00	1.00	1.00	1.00
Sharpe ratio	0.732	0.747	0.767	0.578	0.656	0.562
Total turnover	4.95%	3.23%	17.25%	3.88%	10.82%	26.24%

In our second variation, case B, we introduced a greater degree of heterogeneity among investors in terms of their risk tolerance. Whereas in the base case we assumed that 70% of investors have a risk tolerance between 0.4 and 0.6 before the crisis, here we assumed a uniform wealth distribution across all clienteles (i.e., each clientele is assumed to have one-seventh of the aggregate wealth before the crisis). As shown in the third column of Table 9, risky asset returns and Sharpe ratios are higher in this case than in the base case. The higher demand for safe portfolios from the increased number of risk-averse investors is not quite offset by the risk-bearing capacity of the increased number of risk-tolerant investors.[19] In this variation, the amount of turnover due to tactical responses to the crisis is greater than in our base case (17.25% versus 4.95%) and is driven by the fact that greater heterogeneity in risk tolerance means that more trading is needed to distribute risk optimally.

In our third variation, case C, we assumed that individuals do not become less risk tolerant because of the crisis. In other words, we did not decrement each clientele's risk tolerance by 0.1 as we did in the base case. Note that even in this case, average risk tolerance decreases in the crisis, but only from 0.5 to 0.485 and purely because the more risk tolerant lose relatively more wealth going into the crisis than the less risk tolerant. As expected, post-crisis risk premiums on risky assets do not increase as much as in the base case, and the Sharpe ratio increases by much less than it does in the base case. This result suggests that a key issue in inferring how risk premiums change in a crisis is the degree to which investors become more fearful and less tolerant of risk. We should also emphasize that in all these variations, we are assuming a 40% decline in equities and a 10% decline in bonds. We know that some of the decline in value in a crisis will be due to downward revisions in cash flow projections and some to increases in risk premiums. Thus, in case C, where less of the decline can be attributed to increases in risk premiums, more must be attributed to downward revisions in cash flow projections. Note that in case C, we again see that the tactical responses to the crisis are not significant, with aggregate turnover less than in the base case.

In our next variation, case D, we assumed that 25% of investors in each clientele follow a strategy in which they rebalance their portfolios to maintain the "target weights" they had before the crisis. Obviously, it is impossible for all investors to rebalance to their pre-crisis allocation weights in this way, but a subset of investors can do so. Investors who follow a fixed-weight rebalancing strategy wish to buy equities (which have fallen most in value) and sell bonds and cash, which have fallen by less or, in the case of cash instruments, appreciated. Sharpe (2010) pointed out that those who follow such a target-rate policy are acting as *de facto* contrarians with respect to their allocations. If, for example, the policy weight for equities is 60% and equities drop in value relative to fixed income and other assets in the mix, the target weight plans will be buying equities to get back to the 60% weight. There is no reason to believe that the optimal solution is a policy that always involves rebalancing to a set of fixed weights established at some arbitrary time. In this scenario, the fact that 25% of the investor population is following a contrarian strategy means that risk premiums do not increase as much as they do in the base case. As can be seen in Table 9, risk premiums on equities are about 3% less than they are in the base case because of contrarian trading. The contrarians act as if they have become more risk tolerant and are willing to absorb some of the net supply of risky assets offered by the risk intolerant without needing to be compensated by significantly higher expected returns.

In all the variations we have considered thus far, trading is primarily motivated by differences in risk tolerance. However, it is quite likely that a major explanation for much of the trading observed in a crisis will be differences in investor beliefs, whereby investors agree to disagree and trade.[20] Two investors with the same level of risk tolerance (and the same tolerance for illiquidity, the same untraded asset positions, the same tax positions, and other characteristics that we have not considered) may still find a reason to trade if they have different beliefs about future returns. In a crisis, these different beliefs are more likely to be about macro or systematic factors that affect the performance of entire asset classes than about company-specific fundamentals. But trading motivated by differing beliefs is, in fundamental ways, quite different from the trading we have considered so far. A trade in which a less risk-tolerant investor reduces risk by trading with a more risk-tolerant investor can be objectively viewed as a win-win transaction: Both parties gain because the transaction results in a better sharing of risk. The same case cannot be made as easily for transactions based on differing beliefs. Behind such a transaction is the belief by each party that the other party is on the wrong side of the trade. Objectively, both parties cannot be right. One can trade on one's beliefs, but in doing so, one should remember that even here the laws of supply and demand hold. Essentially, anyone trading on the basis of the market being "wrong" must find some other investor who also wants to trade on the basis of the market being wrong—in the opposite direction. In the same vein, most investors cannot rely on the belief that they can "trade faster" than the average investor in adjusting their allocations to cut their losses as a form of risk control in a crisis scenario.[21]

To analyze how diverse expectations after a crisis might affect our results, we looked at a scenario in which investors do not differ at all in their risk tolerance levels but do differ in their expectations. Specifically, we assumed that *all* investors initially have a risk tolerance level of 0.5 and that this level remains unchanged in the crisis. We also assumed that these investors have the same expectations before the crisis. Investors differ only in their expectations after the crisis. The most optimistic (5% of the market) expect equities to have returns 9% above the market consensus and bonds to have returns 6% above the consensus. The most pessimistic have the opposite forecasts. Our assumptions about expectations are set out in **Table 10**.

Column E in Table 9 shows that divergence in expectations differs from the other cases mainly in the much higher level of turnover that it creates. The risk premiums and Sharpe ratios shown in column E are quite similar to those shown in column C. Because we are keeping risk tolerance unchanged in column E, it is appropriate to compare it with column C.[22]

Conclusion

We have analyzed the tactical shifts in asset allocation that various investor clienteles should make in response to a crisis, imposing the requirement that these shifts be consistent with market equilibrium (i.e., the laws of supply and demand). The crisis scenarios we examined approximately match, at least in their severity, what we experienced in the financial crisis of 2007–2009. We looked at allocations at the level of broad asset classes—namely,

Table 10. Assumed Distribution of Expectations Relative to Consensus across Investor Groups with Identical Risk Tolerances

Clientele	1	2	3	4	5	6	7
% of total wealth	5.00	10.00	20.00	30.00	20.00	10.00	5.00
Deviation from consensus							
Bonds	–6.00%	–4.00%	–2.00%	0.00%	2.00%	4.00%	6.00%
Equities	–9.00	–6.00	–3.00	0.00	3.00	6.00	9.00

U.S. equities, developed-market equities, emerging-market equities, bonds, and cash. In the crisis scenarios we considered, equities were assumed to lose 40% of their value and bonds 10%. We assumed that asset volatilities and correlations across asset classes increase substantially in a crisis and—in most of the scenarios—investors become less risk tolerant.

When investors have suffered losses approaching 40% of their wealth, when investment uncertainty has increased substantially, and when the ability to reduce risk through diversification has diminished, the natural inclination for most investors is to flee to safety by selling off risky assets and replacing them with safe assets. Unfortunately, not everyone can "flee to quality." Any investor who wants to flee to safety must find another investor who is willing to take the opposite side of the transaction and "flee to more risk." Prices must fall to the point where the rewards for bearing risk are high enough for the demand for risky assets to equal the supply. In our base-case depiction of a crisis, we found that equity and bond risk premiums must dramatically increase to bring about this result. An investor's appropriate tactical allocation response to these changes depends critically on that investor's risk tolerance relative to the risk tolerance of the average investor. Investors who are more tolerant than the average investor generally increase their holdings of risky assets, accommodating the demands of the less risk-tolerant investors who wish to do the opposite. This rebalancing occurs notwithstanding the fact that the more risk-tolerant investors experience a greater "hit" to their wealth.

One of our key observations is that the appropriate tactical responses for most investors in a crisis can actually be rather small, especially when the extremely high- and low-risk-tolerant investors do not make up a big portion of investors. In our base case with no differences in investor expectations, we found that for 80% of the investors, the appropriate adjustment involves less than 4% turnover. In the base case, only investors who are extremely risk averse or risk tolerant will find it appropriate to make significant changes in their allocations. If we introduce grounds for more aggressive responses by investors in the form of differences in their expectations in the crisis scenario or if some investors follow a target weight allocation policy that induces extra trading "on autopilot," then turnover will naturally be higher. Also, if there is much greater heterogeneity among investors in their risk tolerances, there will be a higher demand for trading between very risk-tolerant and very risk-intolerant investors—again, with increases in turnover. But if there are leverage restrictions or no change in risk tolerances across investors, turnover is even lower than in our base case.

Our analysis of asset allocation changes is "macroscopic" and thus closer to a starting point than an end point. It does not purport to capture all possible motivations for subsets of investors to adjust their portfolios, especially after a severe dislocation such as what we experienced in the last crisis. For example, in the 2007–09 crisis, liquidity was a major concern for many investors. Some of the observed trading in the market was no doubt due to a subset of investors who needed liquidity selling liquid assets to generate needed cash; the needed liquidity was likely due in part to the "breakdown" of liquidity in other markets in the face of asymmetric information and counterparty risk.[23] However, our general results for the allocation of risky assets in the economy in the face of increasing risk can be carried over to a consideration of how illiquid assets are allocated after a major shock. Just as risky assets became more risky in the 2007–09 crisis, there was a sense that illiquid assets became more illiquid. And just as there was an overall reduction in risk tolerance as investors became poorer and more cognizant of risk, there was in all likelihood an overall reduction in tolerance for illiquidity, which meant that just as risk premiums rose during the crisis, illiquidity premiums most likely rose as well. And just as there is an inclination for investors to flee to safety when risk increases, there is an inclination for investors to flee to liquidity when illiquidity becomes an even greater concern. Of course, the same equilibrium principle that applies to risk also applies to illiquidity: An investor who wants to flee to liquid assets can do so only if some other investor is motivated to "flee to illiquidity." This result will come about only if the illiquidity premium on illiquid assets increases sufficiently to equate the supply and demand of illiquid assets. Just as we had clienteles based on risk tolerance, there will be clienteles based on tolerance for illiquidity. Whether an investor is buying or selling illiquid assets depends on that investor's tolerance for illiquidity relative to the societal average.

Also, beyond differences in the illiquidity of assets, most investors—both households and institutions—have nontraded, completely illiquid assets (including human capital for household investors and opaque "alternative" asset holdings of such institutional funds as university endowments), and the risk–return characteristics of these investors no doubt changed in the crisis. Heterogeneity in exposures to these nontraded assets and in the ability of institutional plan sponsors to meet their liability commitments could provide some further motivations for changing asset allocations. Of course, these changes could not be unidirectional across all investors; obviously, one cannot argue that because of the increased risk of human capital, all investors should underweight equities. Once again, the supply and demand principle must be observed: Any investor who is making an allocation change because of human capital risk or nontraded asset risk must find another investor willing to make the opposite change. Trading will again be based on how the particular investor differs from the "average." Another motivation for adjustments in asset allocations in crises may be related to taxes. Adjustments in response to changes in valuations and risk exposures may differ depending on whether an account is taxable, which may create trading between the taxed and the nontaxed.

We have intentionally kept the examples used in our analysis as simple as possible to illustrate the fundamental observations we wish to make. Much more complicated models could be developed to capture in more detail some of the effects we considered, but the overall message would still be the same: Any tactical portfolio adjustments that investors wish to make in response to changed market conditions take place in a market where the laws of supply and demand govern, and tactical responses must be developed with that in mind. In a crisis, prices and risk premiums must adjust so that, as a rough approximation, one can say that the "average investor" will not want to trade. The trades that any particular investor will want to make depend on how that investor's risk preferences and other characteristics compare with those of the average investor. As we have shown, tactical adjustments will be rather modest for most investors, even for large changes in market conditions, because much of the adjustment to new conditions comes through changes in prices and risk premiums.

We thank Mark Wolfson for helpful comments.

This article qualifies for 1 CE credit.

Notes

1. The increase in correlations occurred both between asset classes and within asset classes. For example, the contribution of common factor risk to the volatility of the typical U.S. stock's return roughly doubled from 30% as late as April 2007 to 60% in early November 2008 (source: Quantal International, www.quantal.com).
2. Risk tolerance can be considered a measure of an investor's *risk appetite* or *risk sentiment*.
3. Turnover would be much lower, by definition, if the market froze as part of a complicated process of market adjustment in a crisis in which illiquidity is important. We discuss these issues later in the article, but to give them their appropriate due would take us far afield from the point of our study (for an excellent recent discussion, see Bhattacharya, Chabakauri, and Nyborg 2012).
4. From 1 January 2000 to 12 September 2008, the realized volatilities on the S&P 500 Index, the MSCI EAFE Index, the MSCI Emerging Markets Index, and the Lehman Aggregate Bond Index (which is currently called the Barclays Capital Aggregate Bond Index) were 18.11%, 16.02%, 18.07%, and 4.25%, respectively.
5. Lee, Marsh, Maxim, and Pfleiderer (2006) found that the correlation between U.S. equity returns and bond returns, which was, on average, negative over 2000–2008 (measuring bonds by the Lehman Aggregate Bond Index), does in fact vary over time (e.g., it was strongly positive in the late 1990s).
6. The model that we used in our study for asset allocation decisions is the standard Markowitz mean–variance model, but we believe that our points would continue to hold in more sophisticated dynamic allocation models or in a state-space solution framework, such as that developed in Sharpe (2007). Our calculations here implicitly assume a setting in which asset return variances and covariances effectively capture most of what is relevant in assessing risk. The most common justification for focusing on second moments in the (conditional) return distribution is that asset returns are, to a close approximation, (multivariate) Gaussian. Much of the post-crisis criticism of financial models has focused on the "fat-tailed," non-Gaussian behavior of returns in the crisis, which will no doubt lead some to question this Gaussian assumption. However, as explained in Marsh and Pfleiderer (2012), shifts in the underlying uncertainty, as proxied at the S&P 500 level by the CBOE Volatility Index (VIX), account for a significant amount of the fatness in the tails of the *unconditional* distribution of returns on the S&P 500. Thus, so long as the increases in the volatilities and covariances of returns assumed in our example are taken to represent the adjustments made in *conditional* forecasts, our use of a Gaussian assumption for these *conditional* returns is not unreasonable.
7. Later in the article, we consider a variation on the base case in which investors' risk tolerances are equal but their expectations differ.
8. Risk tolerance determines the risk penalty an investor assesses for a particular portfolio. An investor with risk tolerance equal to ρ assesses a risk penalty equal to the variance of the portfolio's return (i.e., the square of the volatility) divided by 2ρ.
9. Of course, the actual distribution of investors' risk tolerances is not easily observed. We believe that the distribution of portfolio allocations produced by our assumed distribution of risk tolerances (Table 2) is roughly consistent with the distribution of portfolio allocations observed in practice, which leads us to conclude that our assumed distribution of risk preferences is reasonable for purposes of our illustrative examples.
10. For example, Bateman, Louviere, Satchell, Islam, and Thorp (2010, p. 26) reported that "overall, comparing [survey] results between the relatively tranquil asset market conditions of early 2007 and the full-blown financial crisis of late October 2008 suggests a mild moderating of risk tolerance [by Australian individual retirement fund defined contribution investors]." Note that if capital markets are segmented across countries for whatever reason and Australian investors hold predominantly Australian-domiciled stocks, the

survey's finding as to risk tolerance shift may have been muted because the Australian financial crisis was milder than depicted in our crisis scenario. We know of no survey evidence for risk sentiment in U.S. or European markets during the crisis.

11. Wilcox (2003, p. 62) pointed out that if the average individual's investment policy maximizes expected log returns (which is approximately the same as maximizing risk-adjusted expected returns, as we do here), then a drop in risk tolerance from 0.5 to 0.4 could be interpreted as an increase in the average individual's *discretionary wealth* leverage from 1.0 to 1.2, where "discretionary wealth is the amount one could afford to lose without suffering whatever one defines as a shortfall disaster"—assuming that the shortfall level itself stays constant.
12. From 15 September 2008 to 31 December 2008, the realized volatilities on the S&P 500, the MSCI EAFE Index, the MSCI Emerging Markets Index, and the Barclays Capital Aggregate Bond Index were 62.32%, 53.96%, 60.35%, and 18.20%, respectively. Over this period, the VIX ranged from 30.3% to 80.9% and averaged 55%. Thus, our assumed volatilities are in the range that captures the additional risk created by the events that unfolded in the last quarter of 2008.
13. Our assumed correlations are close to the realized correlations from 15 September 2008 to 31 December 2008. Note in particular that over the crisis period, the realized returns on bonds, which were negatively correlated with the realized returns on equities before the crisis, became positively correlated with the realized returns on each of the equity asset classes.
14. In the financial crisis of 2007–2009, the term structure of the implied volatilities on S&P 500 options was downward sloping, with near-term (less than 30 days) volatilities at times well in excess of 60% and medium-term (30–90 days) volatilities between 35% and 45%. This result suggests that investors at the time were predicting that the extreme uncertainty caused by the crisis would not persist indefinitely but would be at least partly resolved in a relatively short time.
15. Recent evidence in Goetzmann, Watanabe, and Watanabe (2012) suggests that procyclical stocks do indeed tend to have higher average returns over time.
16. This amount is measured by summing the absolute values of all the entries in Panel D of Table 6 (change in % holdings in economy) and then dividing by 2. In dividing by 2, we do not double-count by registering both a purchase and a sale.
17. Consistent with our measure of total turnover, we measured turnover for individual investors as the sum of the absolute value of all transactions divided by twice the portfolio value, or equivalently as the value of all purchases divided by the portfolio value.
18. It should be noted that we focused in our study only on tactical portfolio-rebalancing decisions that investors make after a market shock. Kimball, Shapiro, Shumway, and Zhang (2011) also considered an example of portfolio rebalancing after a stock market crash, but they examined such decisions in the context of an overlapping-generations model, in which investors make savings and consumption decisions jointly with portfolio composition decisions. Like us, Kimball et al. emphasized the importance of market clearing in portfolio rebalancing, but their emphasis on equilibrium considerations led them to focus more on long-term adjustments as opposed to the tactical and more immediate adjustments that we consider here.
19. Technically, the risk tolerance parameter is in the denominator of the expected return equation, and so the result is related to Jensen's inequality.
20. We distinguish the "agree to disagree" differences in beliefs from situations in which investors are asymmetrically informed and less-informed investors face potential losses from trading with the better informed. Asymmetric information can result in less liquidity and less trading in markets and can even lead to a complete market breakdown. This scenario is in contrast to differing beliefs, which tend to increase trading.
21. Indeed, it is estimated that in recent "normal times," some 70% of trade volume in the United States was due to high-frequency trading (HFT), which has more to do with uneven advances in trading technology than with rebalancing demands. There is also some evidence that HFT pulls back in fast-moving "crisis" markets. But we have no market microstructure level of granularity in our analysis, which is why we should not try to compare turnover due only to rebalancing with trade volume due to all possible causes.
22. Note that there is a difference between columns C and E in the pre-crisis case. This difference arises because we are assuming that there is no variation in risk tolerance in the differential expectations case, which slightly alters the equilibrium returns relative to those in the base case or column C.
23. Moreover, when brokers and intermediaries who play a role in equating demand and supply begin to be constrained in an illiquid crisis scenario, the evidence is that they "step back," and the ripple effects of that action amplify the shifts in investor liquidity exposure across the clienteles.

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